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Prime Number Identification in Idiots Savants: Can They Calculate Them?

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Several idiots savants who where able to identify prime numbers have been reported. This ability requires complex calculations, because no simple algorithms are known for determining primes. Many savants, however, who demonstrate this ability, do not possess the arithmetical skills to perform such calculations. Explanations offered for the feats of idiots savants are reviewed in the light of their applicability to the cases of prime identification. Existing models cannot fully explain prime number identification for savants with weak arithmetical skills. The author shows that through the natural tendency of visual perception to be organized symmetrically, a distinction between prime and nonprime numbers can be made. This process could both explain the origin of the interest and the ability to identify prime numbers in mathematically weak savants.

Since the first cases were described by Down (1887) and Binet (1894), the savant syndrome has always remained somewhat of a mystery. Investigators have not been able to explain satisfactorily the existence of isolated prodigious abilities in subjects with a low general intelligence. Hundreds of savants with different kind of skills have been reported and described over the years (see Hill, 1978, or Treffert, 1988, for two good reviews). Skills that have been found are artistic ones, musical abilities, specific number operations, calendar calculating, and memorizing. In many cases, even trained professionals cannot match the savant's performance (e.g., Herme-

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lin & O'Connor, 1990). Most of these reports have been largely descriptive, but the focus has shifted recently to understanding the mental operations through which savants achieve their remarkable feats. Thus far, a unifying explanation, which can explain all the different idiots savants' abilities, has not been found. In fact, research indicates that even savants demonstrating the same skill use different strategies (Hermelin & O'Connor, 1986; Howe & Smith, 1988; Treffert, 1988).

PRIME NUMBER IDENTIFICATION BY IDIOTS SAVANTS

A relatively rare ability found in idiots savants is the recognition or production of prime numbers. It is one of the most intriguing abilities found in savants because of its mathematical complexity. A prime number is a number that cannot be divided by any integer but by itself and the number 1. The number 14 for example is divisible by 2 and therefore not a prime number. The number 13 can only be divided by 1 or by itself and thus is a prime. Few integers are primes (see Table I) and all nonprimes can be arrived at by multiplying primes with each other. Primes, however, stand on themselves; they cannot be made by multiplying two other integers. The Greek Eratosthenes in the 3rd century B.C. used this principle to devise a method for discovering prime numbers: By multiplying all the known primes by 2, 3, 4, and so forth he could eliminate all nonprimes thus sieving out new primes. So far mathematicians have not been able to find a rule to predict directly if a number is a prime or not. The only secure method available to determine if a number is a prime or not is simply trying to divide it by all integers smaller than its square root. If no divisor is found the number is a prime number. This trial-and-error method becomes a very time-consuming task as numbers get larger, and is nowadays left to supercomputers, often as a task to fill up their spare time.

Table I. Number of Primes in Various Intervals

Interval	No.
1–10	5
1–100	26
1–1,000	169
1–10,000	1,230
1–1,00,000	9,593
1–1,000,000	78,499

Notwithstanding the complexity and amount of calculations that would be required to identify or generate a prime, quite a few mentally retarded people, some of them autistic, who often could do so instantaneously, have been reported.

Dase was a prodigious child whose general intelligence was very low and completely lacked mathematical insight (Myers, 1903; Sacks, 1985). As Myers put it "Dase was singularly devoid of mathematical grasp" (p. 83 [310]). In 12 years he made tables of factors and prime numbers for the 7th and nearly the whole 8th million. Dase retained this ability throughout his life.

The Parks (C. C. Park, 1967; D. Park & Iouderian, 1974) described how their autistic daughter at a certain age became fascinated with numbers, especially with primes. She started to systematically factorize numbers and at the age of 13 she knew the prime factors of all the numbers from 1 to 1000 and beyond, and was able to tell, at a glance, how many ways there are of factoring a number. After some years Emily lost her interest in numbers.

Probably the most famous prime identifiers were the autistic twin savants studied by Horwitz and others (Horwitz, Kestenbaum, Person, & Jarvick, 1965; Horwitz, Deming, & Winter, 1969), and later by Sacks (1985). They spontaneously involved themselves in a game of exchanging 6-figure prime numbers and when prompted even 12-figure primes. (They also produced 20-figure numbers, but these could not be verified at the time.) Smaller numbers would be produced within seconds, a 10-figure prime took about 5 minutes. Horwitz et al. (1985) found that the twins were not able to correctly add, subtract, divide, or multiply single-digit numbers. This was partly contradicted, however, in a second article by Horwitz et al. (1969), when they concluded that one of the twins could add up 3-digit numbers. Sacks observed that they did not have any understanding of multiplication or division. At a later age, the twins were separated and subject to therapy, resulting in a loss of most of their savant feats.

Sacks (1985) occasionally mentions two other young autistic people with prime number skills. One compulsively wrote pages full of lists of exclusively prime numbers. The other would tell when prompted with the word "special" if a 4-digit number was a prime, or what were its factors. No additional information on these two subjects is available.

In the only experimental study of this kind of savant by Hermelin and O'Connor (1990) and later by Andersen (1992), a 20-year-old autistic male was studied who was able to recognize and generate 5-digit prime numbers within seconds, with a varying degree of correctness. Response time increased with larger numbers. The subject was able to factorize, add, subtract, multiply, and divide large numbers.

EXPLANATORY MODELS FOR IDIOT SAVANTISM

Explanations offered for this particular phenomenon are extremely scarce and reports on these subjects have been largely descriptive. Some attempts have been made, however, and these are now reviewed together with the explanations offered for the savants syndrome in general, and their applicability to prime identification.

Social Isolation and Compensation

It has been suggested that the social isolation found in autistic people leads to boredom, intense concentration and memorizing, or obsessive preoccupation with obscure facts like dates, bus routes, timetables, lottery numbers, and so forth. The performance of their wizard tricks is often one of the few channels to the social world and thus may be strongly reinforced. Howe and Smith (1988), referring specifically to the skill of calendar calculating, remarked: "A person of normal intelligence would find it extremely difficult, if not impossible, to devote hours to acquire the knowledge possessed by the most successful mentally retarded calendar calculators, largely because so many other things are interesting" (p. 381). These general characteristics of mentally retarded and autistic people can contribute to the understanding of especially motivational factors involved in the acquisition of savant skills. But precisely because these are general characteristics, they do not explain why some are savants and others are not.

Memory

The most commonly found explanation is that the savant relies on extraordinary (eidetic) memory. A memory-based model has the advantage of explaining the high answering speed and would be consistent with the low general intelligence of the savants as it does not require any additional complex mental operations. There exist, however, three objections against the rote memory model (Hermelin & O'Connor, 1990; Spitz & La Fontaine, 1973). First, savant skills often have a sudden onset and do not seem to improve over time. Second, response time in prime identification increases with larger numbers. Third, memory can at most be a contributing factor in cases where savants who identify prime numbers are reported not to have had access to prime tables and in some cases even went beyond existing tables (Myers, 1903; Sacks, 1985).

Calculations and Algorithms

Led by the unsatisfactory memory explanation, authors have looked for more intelligent strategies that might be applied by prime-identifying idiots savants, like eliminating easily identifiable nonprimes. Hermelin and O'Connor (1990), in one of the few controlled experimental studies on prime calculation, found evidence that their subject made use of such a strategy. Both a control subject of high intelligence and the autistic subject, who was able to perform divisions and multiplications of rather large numbers, seemed to use a similar strategy of eliminating numbers divisible by 3 and 11. Both showed a considerable amount of inclusion errors as might be expected of an incomplete exclusion procedure. The use of this incomplete sieve of Eratosthenes was confirmed by a second study on the same subject by Anderson (1992), who found that response times fitted much better an Eratosthenes-based elimination model than a memory model.

An original model proposed by White (1988) showed that by knowing the addition series a number belongs to, a conclusion about being prime or not can be derived. The addition series of 3 for example, would look like 3, 6, 9, 12 and so forth. This can easily be illustrated with the multiplication tables of 10*10, with which most people are familiar. We can instantaneously tell that number 31 does not exist in these tables, thus allowing it to be distinguished as a prime. Such a model would describe very well the daughter of the Parks who had prime number capacities within a 3- to 4-digit range. She systematically determined all factorizations and memorized them.

STRATEGIES WITHOUT CALCULATING

The main objection against the proposed algorithms is that they imply a considerable fluency at mental arithmetic. Savants with mathematical skills can distinguish primes through the discovery that they cannot factorize them, but how can mathematically weak savants distinguish a prime number? Quite a few of the mentioned individuals that perform these feats lack the intellectual capacity to understand multiplication or division, let alone the concept of prime numbers. Yet they are somehow able to distinguish them as being different or special. Except in prime tables, this information is not readily available, neither are prime numbers an observable physical property.

It has to be concluded that these mathematically weak savants apply a strategy that allows them to perform their skills without making complex calculations, at a speed that exceeds a calculation procedure. Except for rather speculative ideas such as computer-like mechanism (Horwitz, *et al.*, 1965), subliminal product (Myers, 1903), or some iterative process or centers for calculation (O'Connor, 1989), which rename the issue rather than explain it, only Howe and Smith (1988) have come up with a serious attempt to solve this problem. In a study of a calendar-calculating subject with limited arithmetical skills, they proposed that calendrical calculators arrive at conclusions through the imaginary manipulation of mental images that represents a calculation: "It is conceivable that this system (manipulating images) enabled him to represent certain computational operations, or the results of them, in image form" (p. 379).

Analogously, a mental image operation can be identified to distinguish a prime number from a normal number, following the particularities of human perception. Early in this century, when the Gestalt tradition developed, it was found that human perception does not conform exactly to physical reality. Things are added or left out of the perception and structure is imposed on the perceptive field. A particular example of structuring perception is the concept of "grouping" (Wertheimer, 1923/1950): Various figures are joined together into a group or a cluster. Characteristics that cause figures to be joined together in a group or cluster are proximity, similarity, and the concept of simplicity or "good figure." What determines a good figure, is not unequivocally definable, but one aspect that certainly contributes to the "goodness" of a figure is symmetry. Wertheimer showed that whenever there is the possibility, there is the tendency to perceive symmetrical groups.

Looking, for example, at six objects with more or less equal properties results, after some time, in the perception of two groups of three or three groups of two. It has to be noted that the perception of symmetrical groups is quite compelling as it is difficult not to see these groups. A prime number of objects does not have such a possibility of forming equal groups and is therefore more likely seen as one group, because dissimilar groups do not form a good figure. Testing for the possibility of the formation of equal groups allows for visually distinguishing a prime number from a nonprime number without any arithmetical operation. Following the natural tendency of grouping, the experience of the difference between numbers that can be divided into equal groups and those that cannot is available to the savant and can explain the origin of this interest.

The activity of looking at something and playing with perception is quite absorbing. Most readers are familiar with this experience of playing with their perception, which we somehow especially engage in during our childhood. Looking at the clouds and discovering animals or faces, or staring at the wallpaper and starting to count or discovering geometrical figures can capture one's attention to the point that it even becomes difficult to stop. The introverted nature of an autistic person may cause such an activity to be as attractive and absorbing as other interests often found in autistic individuals such as bus numbers, dates, weather, and so forth. It is therefore not too speculative to assume that autistic people might be drawn to such an abstract activity as playing with perceptive grouping of numbers of objects. Discovering how most numbers can be divided visually in equal groups, while a select group cannot, gives the latter a special or mysterious quality, which has intrigued scientists for centuries. Spending years playing with these special numbers and looking for new ones finally results in one becoming a real expert in the discovery and recognition of special numbers, unaware of the fact that they represent prime numbers.

For identifying larger numbers, the savant would have to be quite skilled in visually arranging and recognizing numbers. Dantzig (1930) called this ability to recognize the number of a group of objects on sight without counting or making use of symmetrical dispositions: "visual sense for numbers." He claimed that this sense in humans rarely exceeds the number 4. Sacks (1985) reported how both Dase and the twins could recognize large numbers on sight. Dase would say immediately "183" or "79" when seeing beans being poured out of a bag. The twins recognized not only "111" when a box of matches fell on the floor, but also that they were made up of 3 groups of 37. A counting procedure, even in combination with eidetic memory, could not account for the speed with which the answers were given. These two remarkable instances show that some enlarged visual sense for numbers exists in at least some of these savants.

CONCLUSION

A review of the literature on prime number identifying idiots savants suggests that, similar to what was found in relation to other savant skills, different strategies may be used by different savants. For those savants who possess reasonable arithmetical skills, well-trained calculation of algorithms in combination with memorization form a viable explanation for their prime identifying capacity. A strategy of eliminating easy primes fits the error pattern and response times of the mathematically skilled subject of Hermelin and O'Connor (1990). For the Parks's daughter who systematically worked out factors, White's model (1988) of familiarity with addition series seems the most appropriate candidate.

For savants who lack basic calculating skills, a process based on visually analyzing symmetry of numbers allows for identifying prime numbers without calculation and could be at the base of these savants' ability. Formation of symmetries is a process that occurs naturally in the perception of good figures, and as such, could explain the origin of their interest in these numbers. The social reinforcement for their performance and the obsessive quality which is often found in autistic people may carry this skill to great lengths. Anecdotal evidence of an exceptional visual sense for numbers in these weak-arithmetical prime identifiers is consistent with this hypothesis. Indications for visual processing of information, mimicking calculation, were also found in a mathematically weak calendrical calculator.

The proposed model cannot be more than a good hypothesis until it is experimentally tested. Experimental testing of the visual perception of large numbers, and sensitivity to symmetrically and nonsymmetrically arranged objects can reveal more about this process. The limits of the capacity to instantly identify the number of objects should be found to be in the same range as the largest prime numbers that can be classified correctly. The smallest factor of a nonprime, representing the simplest possible symmetry, should determine response time in unmasking it as a nonprime.

Mathematical abilities are generally assessed in studies on savants, but the consequences of these measures are usually not fully appreciated. On the one hand, models relying heavily on calculation are proposed in cases where arithmetical skills are weak. On the other, it is seldom noted that having basic skills like division or multiplication inevitably implies abstract thinking. This is confirmed by a few studies in which mathematically strong savants demonstrate isolated high scores on nonverbal tests requiring abstraction. Steel, Gorman, and Flexman (1984) and Hermelin and O'Connor (1990) found isolated high scores on the Progressive Matrices; Ho, Tsang, and Ho (1991) found an above-average score on the Block Design, which, though principally a test measuring visuospatial organization, has also been related to abstract thinking (Lezak, 1983). To the author's knowledge no such instances have been reported for mathematically weak savants.

This is especially important because it contradicts the idea often defended that savants, and autistic people in general, could be characterized by concreteness and an inability of abstract thinking. These findings suggest that abstract thinking is retained at least in some savants.

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